Abstract No: 199 ***ICT, Mathematics, and Statistics***

ON CONNECTEDNESS PROPERTIES OF COMPLEMENT OF CLOSED HAUSDORFF WEAKLY INFINITE-DIMENSIONAL SUBSET IN THE MODULI SPACE OF ALL COMPLETE RIEMANNIAN METRICS ON THE PLANE

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In Riemannian geometry, introducing geometric concepts to smooth manifolds is done via selecting an appropriate Riemannian metric. We define $R_{\geq 0}(R^2)$ to be the space of all complete Riemannian metrics of non-negative curvature on the plane. The Lie group $Diff(R^2)$ of all self diffeomorphisms onto R^2 acts on $R_{\geq 0}(R^2)$ by pulling back metrics. Denote the moduli space of all complete Riemannian metrics of non-negative curvature on the plane by $M_{\geq 0}(R^2)$, it is the quotient space of $R_{\geq 0}(R^2)$ by the $Diff(R^2)$ action via pullback. The moduli space $M_{\geq 0}(R^2)$ is not a manifold since different Riemannian metrics may have isometry groups of different dimensions. A topological space X is said to be weakly infinite-dimensional if for every family $\{(A_i, B_i): i \in N\}$ of pairs of disjoint closed subsets of X, there exist separators D_i between A_i and B_i such that $\bigcap_{i=1}^{\infty} D_i = \emptyset$. The connectedness properties of the space $R_{\geq 0}(R^2)$ and $M_{\geq 0}(R^2)$ were first studied by Belegradek and Hu, and they proved that the complement of every finite-dimensional subset of the space $R_{\geq 0}(R^2)$ is continuum-connected. It was later proved that the complement of every closed, finite-dimensional subset of $R_{\geq 0}(R^2)$ is path-connected and that the complement of a subset of $M_{\geq 0}(R^2)$ is path-connected if the subset is countable, or it is closed, metrisable and finite-dimensional. The results for $R_{\geq 0}(R^2)$ were generalised to show that complement of every closed, weakly infinite-dimensional subset of $R_{\geq 0}(R^2)$ is path-connected. Further, a partial generalisation on $M_{\geq 0}(R^2)$ was obtained to prove that the complement of a closed Hausdorff space with Haver's property C of $M_{\geq 0}(R^2)$ is path-connected. In this research, we prove that the complement of every closed Hausdorff weakly infinite-dimensional subset of $M_{\geq 0}(R^2)$ is path-connected, with an argument using a dimension theoretic argument on the dimensionality of a paracompact preimage of a fully closed map onto a weakly infinite-dimensional space. With this result, we conclude the series of theorems of connectedness properties of $R_{\geq 0}(R^2)$ and $M_{\geq 0}(R^2)$.

Keywords: Moduli space, Riemannian metrics, Weakly infinite-dimensional